

**Phys 410**  
**Spring 2013**  
**Lecture #11 Summary**  
**15 February, 2013**

We considered two-particle conservative-force interactions and found that the potential function  $U(\vec{r}_1 - \vec{r}_2)$  does double duty by representing two forces:  $\vec{F}_{12}$  and  $\vec{F}_{21}$ . In other words, one can recover the forces of interaction between the two particles as  $\vec{F}_{12} = -\vec{\nabla}_1 U(\vec{r}_1 - \vec{r}_2)$  and  $\vec{F}_{21} = -\vec{\nabla}_2 U(\vec{r}_1 - \vec{r}_2)$ , where the subscripts on the gradients denote derivatives with respect to the coordinates of one of the particles. In addition, this single potential energy function properly keeps track of the total potential energy of the two particles interacting by means of the associated conservative forces. The total mechanical energy of the two-particle system is given by  $E = T_1 + T_2 + U$ , and this quantity is conserved. This treatment can be generalized to an arbitrary number of particles interacting with each other by means of conservative forces, and under the influence of external conservative forces.

We then discussed the physics of oscillations. Any system with a minimum in the potential energy landscape  $U(x)$  can have small harmonic oscillations around the minimum in the potential. The Hooke's law spring constant is just the local curvature of the potential at the minimum:

$k = d^2U / dx^2 \Big|_{x_0}$ . A one-dimensional harmonic oscillator of mass  $m$  obeys the equation  $m\ddot{x} = -kx$ .

Dividing through by  $m$  and defining the natural oscillation frequency  $\omega_0 = \sqrt{k/m}$ , the equation becomes  $\ddot{x} = -\omega_0^2 x$ . This equation can be solved in numerous ways, and the solutions can be written in several canonical forms, including:

- 1)  $x(t) = C_1 e^{i\omega t} + C_2 e^{-i\omega t}$
- 2)  $x(t) = B_1 \cos(\omega t) + B_2 \sin(\omega t)$
- 3)  $x(t) = A \cos(\omega t - \delta)$
- 4)  $x(t) = \text{Re}[A e^{i(\omega t - \delta)}]$

All of these forms can be related to each other, as you will prove in homework.

Finally we considered the energy in simple harmonic motion. The total mechanical energy is  $E = T + U = (m/2)\dot{x}^2 + (k/2)x^2$ . Using form 3 above (for example), this can be written as  $E = (k/2)A^2$ , which is constant. The kinetic and potential energies are both varying with time as  $\sin^2(\omega t - \delta)$  and  $\cos^2(\omega t - \delta)$ , respectively. They both oscillate between 0 and  $E$  twice per period of oscillation, and are exactly  $180^\circ$  out of phase. The shuttling of energy back and forth between two

different forms (in this case potential and kinetic) is a hallmark of simple harmonic oscillation, and resonance.